Massive MIMO Performance with Timing & Frequency Errors

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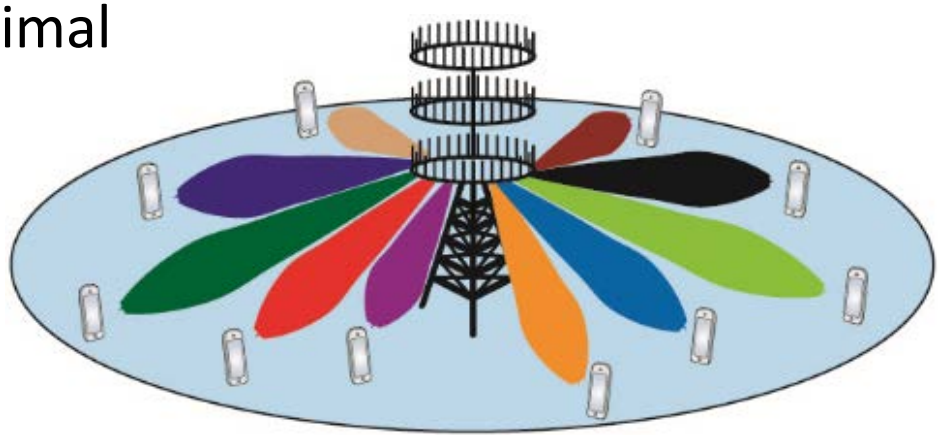
Background and Problem

- Massive MIMO [1]
  - Bring significant spectral efficiency gains
  - Linear precoding is near-optimal
  - TDD reciprocity

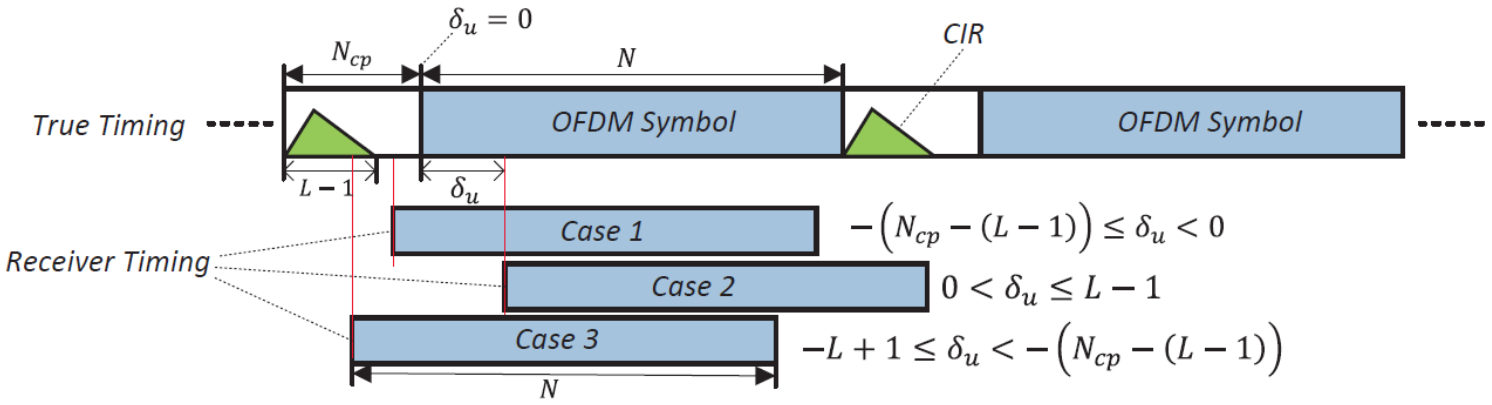


- TOs and CFOs
  - Residual TOs and CFOs exist at the BS.
  - Can residual TOs or CFOs lead to pilot contamination?
  - How to mitigate the pilot contamination?

[1] L. Lu, G. Y. Li, A. L. Swindlehurst, A. Ashikhmin, and R. Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE Trans. Commun.*, vol. 8, no. 5, pp. 742-758, Oct. 2014.

TOs and CFOs

- Residual TOs:



- Residual CFOs: Normalized CFO is

$$\epsilon = \frac{f_c - f_c'}{\Delta f}$$

  - Δf: subcarrier spacing.

System Model

- UL Received Signal [2]:

$$Y_m = \sum_{u=1}^{U} \sqrt{N}\, F E_u F^H \Delta_u S_u F_L h_{m,u} + N_m,$$

CFO and TO of u-th user

where $E_u = \text{diag}\{[1, e^{j2\pi\epsilon_u/N}, \dots, e^{j2\pi(N-1)\epsilon_u/N}]\}$,

$\Delta_u = \text{diag}\{[1, e^{j2\pi\delta_u/N}, \dots, e^{j2\pi(N-1)\delta_u/N}]\}$.

- UL Orthogonal Pilot Design:
  - TDM Pilots:

$S_u = \text{diag}\{[1, e^{-j2\pi\tau_u/N}, \dots, e^{-j2\pi\tau_u(N-1)/N}]\}$, $\tau_u := (u-1)L$

  - FDM Pilots:

$S_u[k,k] = \sqrt{U}$, if $\text{mod}(k+u-1, U) = 0$

[2] Y. Mostofi and D. C. Cox, "Mathematical analysis of the impact of timing synchronization errors on the performance of an OFDM system," *IEEE Trans. Commun.*, vol. 54, no. 2, pp. 226-230, Feb. 2006.

System Model

- UL Channel Estimation:
  - LS-based channel estimation: with TOs and CFOs

$$\widehat{H}_{m,u}[n] = \sum_{k=0}^{N-1} \Psi_{u,u}[n,k] H_{m,u}[k] + \sum_{u' \neq u}^{U} \sum_{k=0}^{N-1} \Psi_{u,u'}[n,k] H_{m,u'}[k] + N'_{m,u}[n]$$

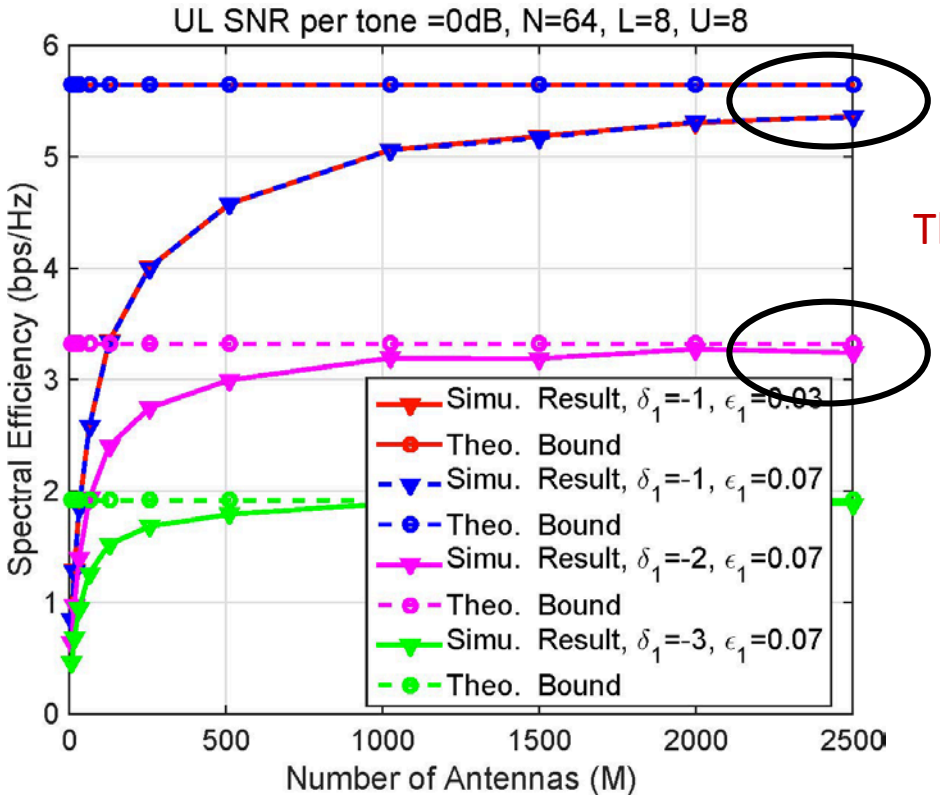
inter-user-interference + noise

- DL Data Transmission:
  - TDD reciprocity, MF precoding

Effect of Residual TOs and CFOs

TDM Pilots

- Residual TOs lead to pilot contamination



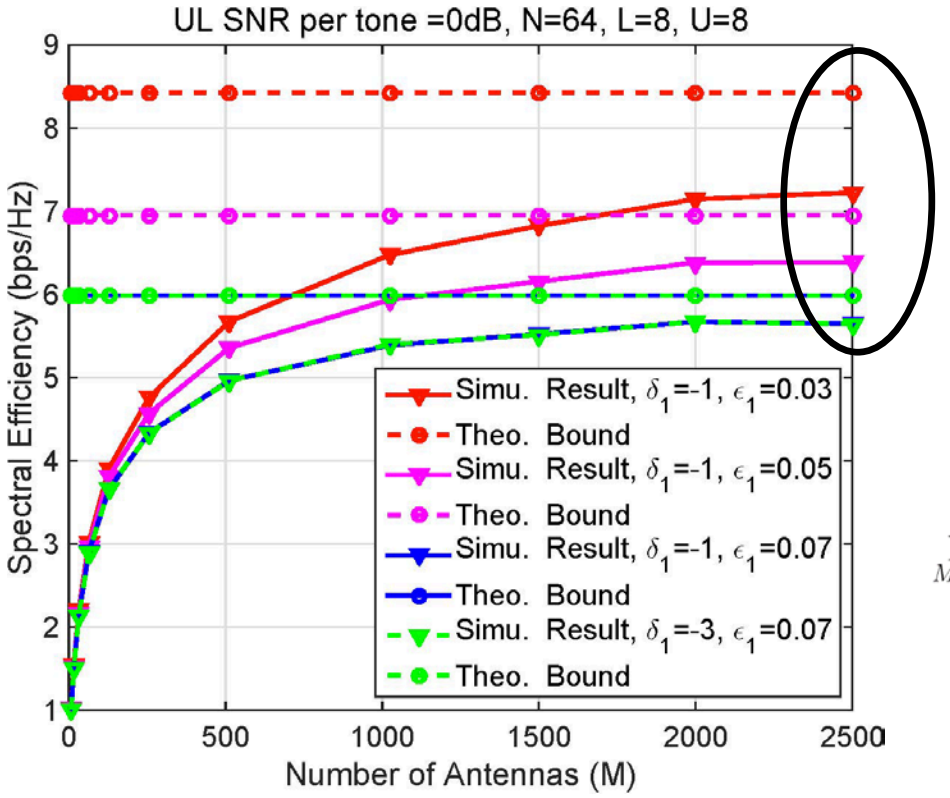
The DL spectral efficiency is bounded

$$R_{u,n}^{LS,\,TDM} = \log\left(1 + \frac{\left|\sum_{l=0}^{L+\delta_u-1} \sigma_{u,l}^2 e^{j2\pi\epsilon_u l/N}\right|^2}{\left|\sum_{l=L+\delta_u}^{L-1} \sigma_{u,l}^2 e^{j2\pi\epsilon_u l/N}\right|^2}\right)$$

Effect of Residual TOs and CFOs

FDM Pilots

- Residual CFOs lead to pilot contamination



The DL spectral efficiency is bounded

$$\lim_{M\to\infty} R_{u,n}^{\text{LS, FDM}} = \log\left(1+\frac{\sin^2(\pi\epsilon_u)/\sin^2(\pi\epsilon_u/U)}{\left|\sum_{u'\neq u}^{U}\sum_{n=0}^{U-1} c_{u',n}c_{u,n}^{*}e^{j2\pi\epsilon_u n/U}\right|^2}\right)$$

Effect of Residual TOs and CFOs

Important Results:

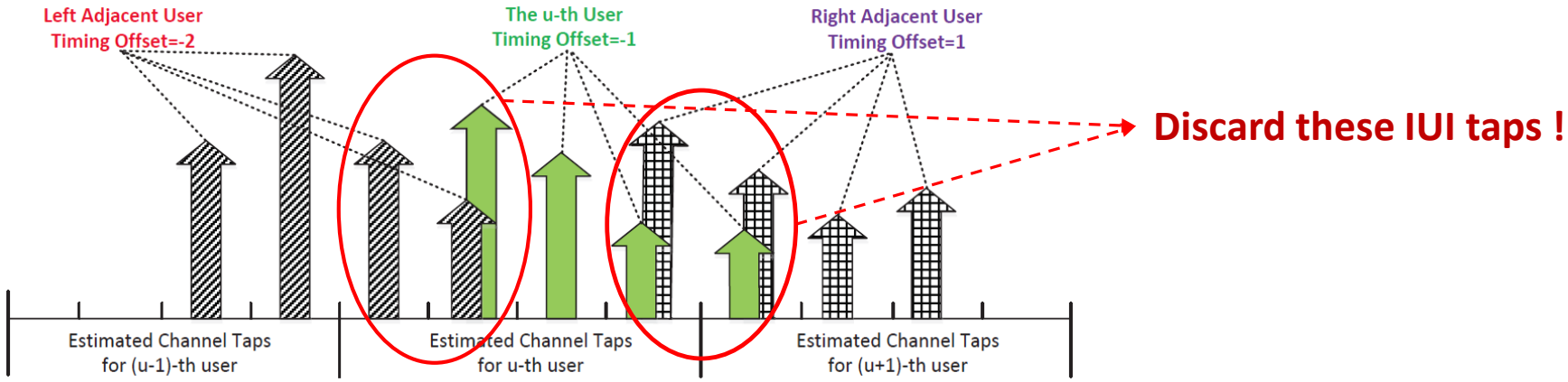
- The DL spectral efficiency of the u-th user is just function of its own residual TO and CFO.
- The FDM pilot design is always robust to TOs, while the TDM pilot design is robust to CFOs.

Pilot Design Principle:

- Significant residual CFOs----TDM pilots
- Significant residual TOs----FDM pilots

Pilot Decontamination

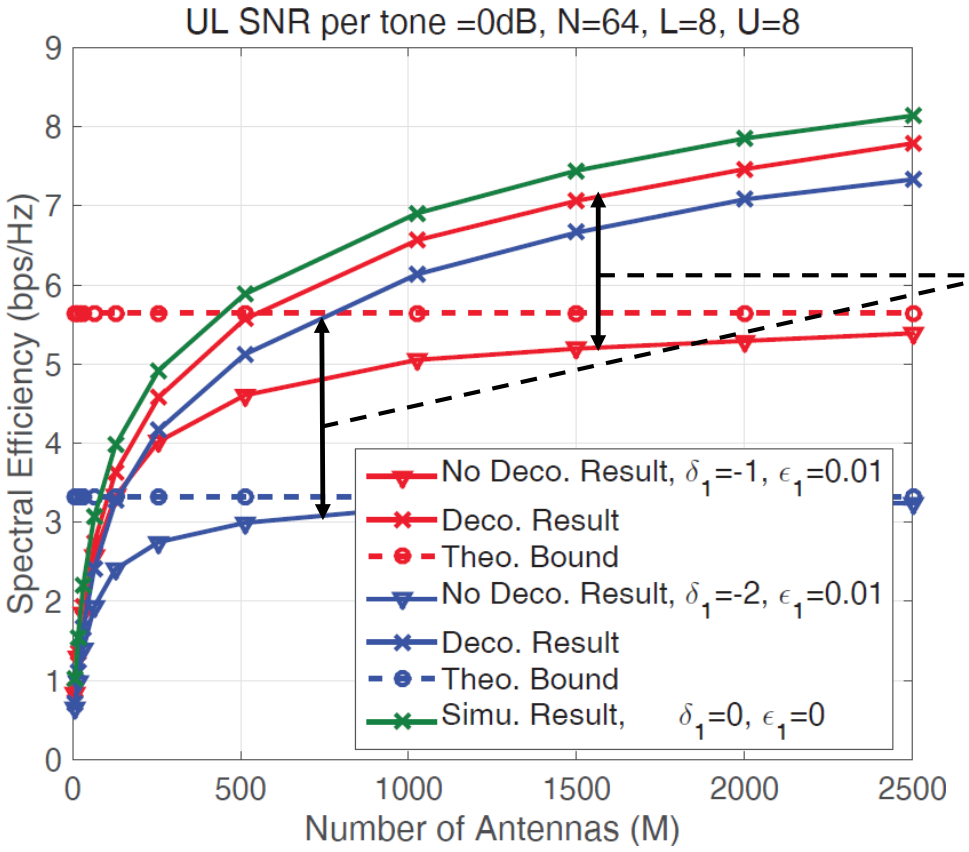
- With TDM Pilots, residual TOs lead to IUI



- Decontamination: Simply discard some taps of the user

Pilot Decontamination

- TDM Pilots: the pilot contamination can be avoided by discarding IUI taps



Performance gain after discarding some estimated taps

Concluding Summary

Residual TOs and CFOs both lead to pilot contamination. We can mitigate the contamination by discarding some estimated taps.

Thanks for your attention!

Questions?

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